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CALCULATION OF DYNAMIC LOADS IN METAL STRUCTURES OF CRANE BOOM

РОЗРАХУНОК ДИНАМІЧНИХ НАВАНТАЖЕНЬ В МЕТАЛОКОНСТРУКЦІЯХ СТРІЛИ КРАНА

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Abstract. *To ensure trouble-free operation and increase the reliability of cranes in the calculations of structures and components of their working equipment, it is important to take into account dynamic loads that are several times higher than static loads. Elements of dynamic loads in the crane suspension are its elastic components (flexible traction organs) - ropes.*

The method of determining the forces in the suspension of the load, the duration of the selection of gaps (tension of the ropes) at the first stage, the speed of separation of the load from the base at the second stage, the maximum force in the elastic element at the third stage makes it possible to significantly simplify the solution of complex equations, to determine simple expressions and with sufficient accuracy for practical calculations to determine their values.

Key words: *crane, mechanism, load, torque, force.*

Introduction.

Loading and unloading operations are an integral part of the construction process. To perform these works, for the most part, cranes of various types are used.

Cranes as lifting machines are widely used in construction to move goods and install structures.

Scientific and technological progress, taking place in all countries of the world, urgently requires an increase in productivity, lifting capacity and an increase in the working speeds of lifting machines, which leads to a reduction in transient processes, that is, to a decrease in the acceleration and braking time of machines.

All this leads to an increase in the intensity of the load-lifting machine, causes additional efforts on all elements of the machine, which received the name in the technique – external dynamic loads.

On the other hand, any machine has structural features of its kinematics, deformability of flexible elements – all this in the process of the machine causes oscillatory processes in the suspension of the load and refers to the phenomena – the internal dynamics of the machine.

For safe operation of cranes, it is important to take into account the magnitude of all types of dynamic loads when calculating their structures and selecting components [1, 2].

Purpose of the study.

Develop a methodology for determining dynamic loads in the crane suspension when lifting a load from a rigid base in order to simplify complex calculations and determine simple expressions for practical calculations.

Presentation of the main material.

Consider the dynamic loads in the lifting mechanism when the load is on a rigid basis when it is lifted [3].

When accelerating the lifting mechanism, when the load is on a rigid base, two cases are distinguished:

1. Start the mechanism with elastic pick-up of the load.
2. Start with pick-up cargo.

Under the start with elastic pick-up is understood the case when the acceleration of the mechanism begins with the load lowered on the support with the tension of the ropes, i.e. $0 < F < G_{van}$, where F – rope tension force; G_{van} – weight of cargo.

Under the launch with the pick-up of the load is understood the case when the acceleration of the mechanism begins when the load is lowered on the support and the ropes are weakened, i.e $F < 0$.

When lifting the load with pick-up, depending on the tension of the ropes, the type of engine and starting system, two cases are considered:

the ropes can begin to stretch until the end of engine acceleration, i.e $M_d < M_n$

ropes can begin to be stretched after full acceleration of the engine, i.e $M_d = M_n$, where M_d is the actual engine torque; M_n – rated engine torque.

Dynamic loads in the second case are much higher than in the first.

When considering dynamic loads in the crane suspension during start-up with pick-up, we first assume that the load is on a rigid base and consider the crane metal structure to be absolutely rigid.

The process of lifting "with pick-up" is considered as a phased [4]:

the first stage is the choice of gaps and tension of the ropes;

the second stage is a reference stage, when the force in the elastic element increases to the value "G_{van}" with a fixed mass "m_2";

the third stage is the post-separation stage, which begins with the movement of the mass "m_2" torn from the support.

At the first stage, the gap «Δ» in this system is selected. During the first stage, the mass «m_1» moves under the influence of a constant average starting force «P_1», while the gap «Δ» is selected (Fig. 1).

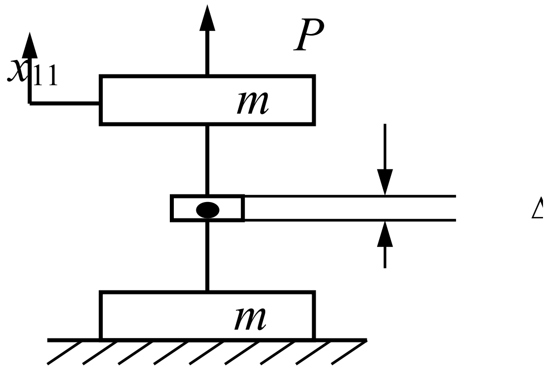

Figure 1 - Diagram of the first stage with cargo pick-up

The equation of motion of the conducting mass, in the first stage, will be written:

$$m_1 \ddot{x}_{11} = P_1 .$$

Integrating twice, we get:

$$x_{11} = \int_o^t \dot{x}_{11} dt = \frac{P_1}{2 \cdot m_1} t^2 ,$$

with x = Δ get the duration of the first stage:

$$t = \sqrt{\frac{2m_1 \Delta}{P_1}} ,$$

where Δ – clearance or value of rope weakening.

Considering accelerated pick-up in the case when the driving force linearly depends on the speed of the driving mass, which is characteristic of the drives of cranes with an asynchronous electric motor or a DC shunt motor. In the first approximation, we can assume that the electric motor at all stages of picking up works on one artificial characteristic (Fig. 2).

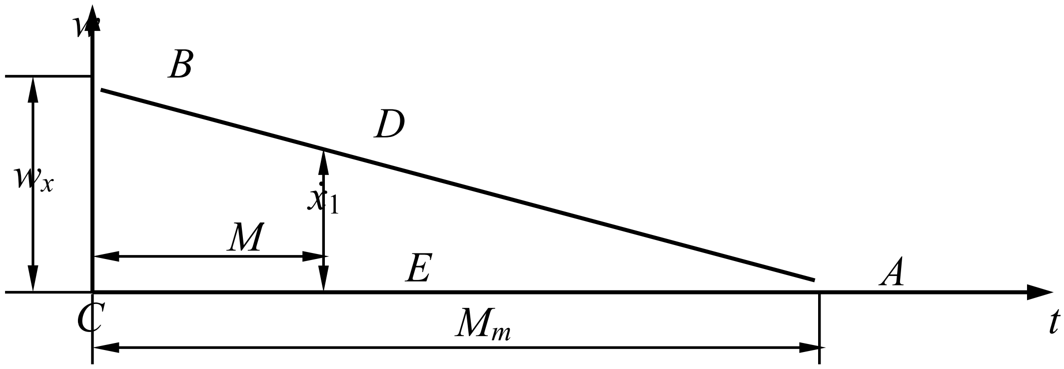

Figure 2 - Simplified artificial characteristic of the electric motor

Here M_m is the maximum engine torque, M – current value of engine torque, w_x – angular speed of engine idling, $\dot{x}_1$ – current value of engine angular speed.

From the similarity of the triangles ABC and ADE, we write:

$$\frac{M_m - M}{\dot{x}_1} = \frac{M_m}{w_x}; \qquad \frac{M_m}{\dot{x}_1} - \frac{M}{\dot{x}_1} = \frac{M_m}{w_x}; \qquad \frac{M_m}{\dot{x}_1} - \frac{M_m}{w_x} = \frac{M}{\dot{x}_1},$$

multiplying by the current speed, " $\dot{x}_1$" we get:

$$M = M_m - M_m \frac{\dot{x}_1}{w_x}; \qquad M = M_m - B\dot{x}_1,$$

where $B = M_m / w_x$ is the proportionality factor.

Consider the case when the gaps are chosen before the operator moves to the next stage of the artificial characteristic of the engine, then the equation of motion in the first stage, the mass «m_1» will be rewritten:

$$m_1 \ddot{x}_{11} = M_m - B\dot{x}_{11}; \qquad m_1 \ddot{x}_{11} + B\dot{x}_{11} = M_m,$$

divide by «m_1» and integrate by «t», finally, we get [5]:

$$m_1\dot{x}_{11} + Bx_{11} = M_m t$$

– is a first order equation.

The characteristic equation of which:

$$\lambda_1 = -B/m_1 = const.$$

In this case, the expression for moving the conductive mass «m_1» will be:

$$x_{11} = A_{12}e^{\lambda_1 t} + A_{13}t + A_{14},$$

at $t = 0$ $x_{11} = 0$ and $\dot{x}_{11} = 0$,

get, $0 = A_{12} = A_{14}$ i.e. $A_{12} = -A_{14}$.

We differentiate this equation, find the coefficients «A_{12}» and «A_{14}»:

$$\dot{x}_{11} = A_{12}\lambda_1 e^{\lambda_1 t} + A_{13}.$$

The first stage ends with a complete selection of the gaps «Δ» in the system [5], i.e

$$x_{11} = \Delta = \frac{m_1 w_x^2}{M_m}\left(e^{\lambda_1 t} - 1\right) + w_x t,$$

decomposing the function into a series (exponent $e^{\lambda_1 t}$)

$$e^{\lambda_1 t} = 1 + \frac{\lambda_1 t}{1!} + \frac{(\lambda_1 t)^2}{2!} + \cdots + \frac{(\lambda_1 t)^n}{n!},$$

where $\lambda_1 = -\frac{B}{m_1} = -\frac{M_m}{w_x m_1}$,

leave the three terms of the decomposition of the equation and shortening, determine the duration of the first stage:

$$\Delta = -w_x t + \frac{M_m t^2}{2m_1} + w_x t,$$

from where, the duration of the clearance sampling (rope tension) at the first stage will be:

$$t = \sqrt{\frac{2m_1\Delta}{M_m}}\,.$$

The second stage is a reference stage, when the force in the elastic element increases to the value «G_{van}» with a fixed mass «m_2» (Fig. 3).

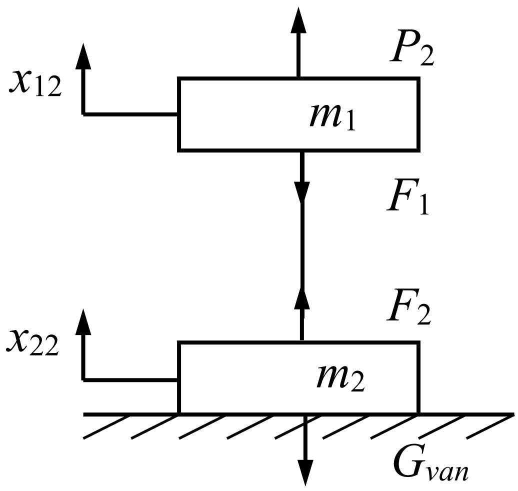

Figure 3 - Diagram of the second stage of cargo lifting

For the initial condition at the second stage we accept, the final condition of the first stage that is «m_1» – has initial speed, «x_{11}» – displacement is equal to zero, «F_1» – the force is equal to zero.

At the second stage, the elastic link begins to load to the value «G_{van}».

The differential equation of mass motion «m_1» will be written:

$$m_1\ddot{x}_{12} = P_2 - F_2\,,$$

since $F_2 = G_{van} + c(x_{12} - x_{22})$, $\omega^2 = c / m_1$, finally get:

$$\ddot{x}_{12} + \omega^2 x_{12} = \frac{P_2}{m_1}\,,$$

where ω – natural frequency of free oscillations of mass «m_1», P_2 – driving force.

If we substitute the values of the force «F_2»and the driving force «P_2» into the differential equation of motion, we get:

$$\ddot{x}_{12} + \frac{B}{m_1}\dot{x}_{12} + \frac{c}{m_1}x_{12} = \frac{M_m}{m_1}\,.$$

Since $\omega = \sqrt{\dfrac{c}{m_1}}$ – the intrinsic frequency of mass oscillation «m_1», $n = \dfrac{B}{2m_1} = \dfrac{M_m}{2m_1 w_x}$ – coefficient characterizing the resistance of the medium, we obtain a second-order differential equation:

$$\ddot{x}_{12} + 2n\dot{x}_{12} + \omega^2 x_{12} = \frac{M_m}{m_1},$$

characteristic equation, will be:

$$x_{12} = \frac{M_m}{c} + \frac{w_x c + M_m \lambda_2}{c(\lambda_1 - \lambda_2)} e^{\lambda_1 t} +$$
$$+ \frac{M_m c + M_m \lambda_1}{c(\lambda_2 - \lambda_1)} e^{\lambda_2 t},$$

if $n > \omega$, then the roots of the characteristic equation are different and real.

In this case

$$x'_{12} = D = const, \qquad \dot{x}_{12} = \ddot{x}_{12} = 0.$$

From the characteristic equation, then we find:

$$D = \frac{M_m m_1}{c\, m_1} = \frac{M_m}{c}.$$

Since the start of the second stage begins at the end of the first:

$$\dot{x}_{12} = A_{21}\lambda_1 + A_{22}\lambda_2.$$

After the movement «x_{12}» reaches such a value that the force in the elastic element becomes equal to the weight of the load «G_{van}», the second stage ends with an equation for determining the movement when the load is torn off the base with a duration «t_2»:

$$x_{12} = \frac{M_m}{c} + \frac{\dot{x}_{12} c + M_m \lambda_2}{c(\lambda_1 - \lambda_2)} e^{\lambda_1 t_2} +$$
$$+ \frac{c\dot{x}_{12} + M_m \lambda_1}{c(\lambda_2 - \lambda_1)} e^{\lambda_2 t_2},$$

$$\frac{G_{van}}{c}=\frac{M_m}{c}+\frac{\dot{x}_{12}c+M_m\lambda_2}{c(\lambda_1-\lambda_2)}e^{\lambda_1 t_2}+\frac{\dot{x}_{12}c+M_m\lambda_1}{c(\lambda_2-\lambda_1)}e^{\lambda_2 t_2}.$$

The third stage is the post-break stage, which begins with the movement of the mass «m_2», which is torn from the support.

At the third stage, both masses in the system have movement (Fig. 4).

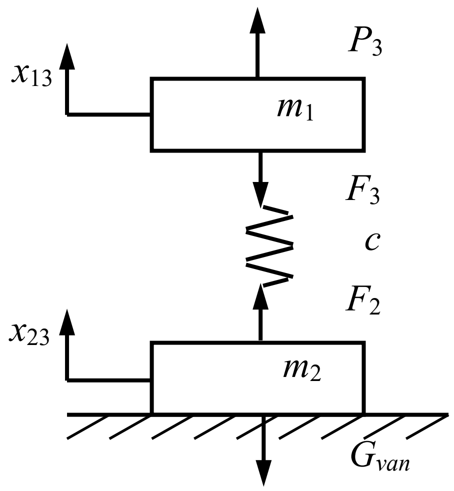

Figure 4 - Stage 3 Diagram

The differential equation of motion of both masses is written:

$$\begin{cases} m_1\ddot{x}_{13}=P_3-F_3 \\ m_2\ddot{x}_{23}=F_3-G_{van} \end{cases}.$$

During this period, the driving force «P_3» differs little from the value of the weight of the cargo, therefore, in the first approximation, we can assume that $P_3 = G_{van}$.

From the differential equation of motion, we find the acceleration values of both masses:

$$\ddot{x}_{13}=\frac{G_{van}-F_3}{m_1};$$

$$\ddot{x}_{23}=\frac{F_3-G_{van}}{m_2}.$$

We differentiate twice the equation of elastic load and substitute the values of mass acceleration:

$$\ddot{F}_3=\omega^2F_3=G_{van}\omega^2.$$

We obtain a linear inhomogeneous second-order differential equation with constant coefficients, where $\omega=\sqrt{c\frac{m_1+m_2}{m_1 m_2}}$ is the frequency of natural mass oscillations in the post-break period.

The general solution of this equation will be written:

$$F_3 = A\cos\omega t + B\sin\omega t + F_3';$$

at the same time we accept:

$$F_3' = D = const\ ,\ \ \dot{F}_3 = \ddot{F}_3 = 0\,.$$

Replacing these values in differential equation, we get:

$$D\omega^2 = G_{van}\omega^2\,.$$

Final force in elastic element [5]:

$$F_3 = \frac{cV_{vidr}}{\omega}\sin\omega t + G_{van},$$

where V_{vidr} – швидкість першої маси у момент відриву вантажу від опори [5].

$$V_{vidr} = \frac{(c\ddot{x}_{12} + M_m\lambda_2)\lambda_1}{c(\lambda_1-\lambda_2)}e^{\lambda_1 t_2} + \frac{(c\ddot{x}_{12} + M_m\lambda_1)\lambda_2}{c(\lambda_2-\lambda_1)}e^{\lambda_2 t_2}\,.$$

Hence it follows that the force in the elastic element after the separation of the load m_2 from the support fluctuates near the value «G_{van}» with amplitude «$\frac{cV_{vidr}}{\omega}$» and circular frequency «ω» (Fig. 5).

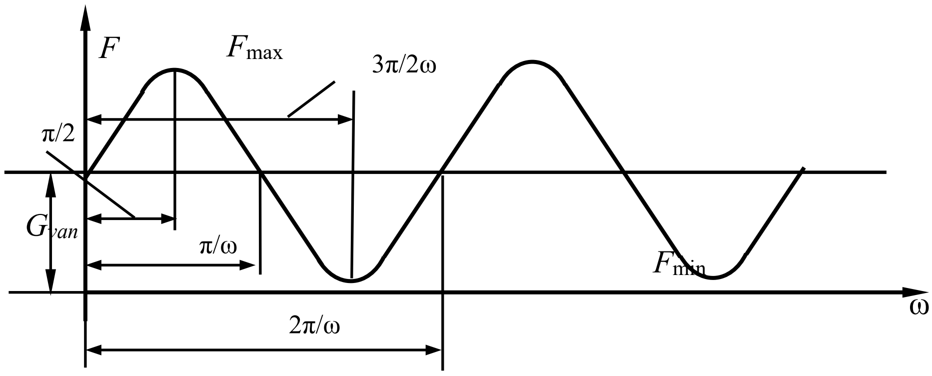

Figure 5 - Graph of the force oscillation in the elastic element after the load is detached from the support

The maximum value of the force in the rope in the post-break period will be at $t = \pi/2\omega$, then:

$$F_{\max} = G_{van} + V_{vidr}\sqrt{\frac{c(m_1 m_2)}{m_1 + m_2}}.$$

From this expression it follows that the dynamic maximum force in the elastic element is directly proportional to its rigidity «*c*» and the speed of the load at the moment of separation from the base.

In many cranes, the kinetic energy of the masses of the rotating mechanism is many times greater than the kinetic energy of the nominal lifting weight, that is $m_1 >> m_2$, especially in general-purpose cranes, in which m_1=(10…20)m_2. For such mechanisms, it can be assumed that $m_1+m_2 \cong m_1$, and when picking up the load from the support, the speed of the mass does not change [6, 7].

Then the maximum force in the elastic element:

$$F_{\max} = G_{van} + V_{vidr}\sqrt{cm_2}.$$

In fact, the value of the maximum force is somewhat less than the value obtained from this equation. Nevertheless, the simplicity and clarity of the physical meaning of this expression allow it to be used in preliminary practical calculations.

Conclusions.

The use of this technique allows you to determine simple expressions when calculating the dynamic load in the crane suspension when lifting a load from a rigid base with sufficient accuracy for practice. This simplifies calculations and reduces their duration.

In the future, it is necessary to develop programs to perform these calculations using computers.

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Анотація. *Навантажувально-розвантажувальні роботи є невід'ємною складовою технологічного процесу будівництва. Для виконання цих робіт здебільшого застосовують крани різних типів.*

Для забезпечення безаварійної роботи і підвищення надійності кранів при розрахунках конструкцій і комплектуючих елементів їх робочого обладнання важливо враховувати динамічні навантаження, які в декілька разів перевищують статичні навантаження. Елементами динамічних навантажень в підвісці крана є його пружні складові (гнучкі тягові органи) – канати.

Розглянуто процес підіймання вантажу з жорсткої основи з його підхопленням, який розділяється на три етапи: перший - вибір зазорів і натяг канатів; другий – довідривна стадія підіймання вантажу; третій - післявідривна стадія підіймання вантажу.

Для кожного етапу прийняті початкові умови, складені диференціальні рівняння руху вантажів, наведено їх розв'язок з урахуванням багатьох чинників і виведені вирази для визначення зусиль в підвісці вантажу, тривалості вибору зазорів (натягу канатів) на першому етапі, швидкості відриву вантажу від основи на другому етапі, максимального зусилля в пружному елементі на третьому етапі.

Наведена в роботі методика визначення зусиль в підвісці вантажу, тривалості вибору зазорів (натягу канатів) на першому етапі, швидкості відриву вантажу від основи на другому етапі, максимального зусилля в пружному елементі на третьому етапі дозволяє значно спростити розв'язання складних рівнянь, визначити прості вирази і з достатньою для практичних розрахунків точністю визначати їх величини.

Ключові слова:*. кран, механізм, навантаження, момент, зусилля.*

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