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A COUPLED NONLINEAR BUCKLING AND PROGRESSIVE DAMAGE MODEL FOR COMPOSITE SHELL STRUCTURES

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Abstract *Thin-walled composite shells fabricated from carbon fiber–reinforced polymer (CFRP) and glass fiber–reinforced polymer (GFRP) are widely employed in aerospace structures due to their high specific stiffness and strength. Their structural response under compressive loading is governed by a strong coupling between geometric nonlinearity, material anisotropy, and progressive damage mechanisms. This paper presents a unified theoretical and numerical framework for modeling nonlinear buckling and progressive damage in laminated composite shells. The formulation integrates von Kármán nonlinear shell kinematics with anisotropic constitutive relations and continuum damage mechanics–based degradation laws. Damage initiation is captured using ply-level failure criteria, while damage evolution is modeled through stiffness degradation. The proposed framework enables the prediction of critical buckling loads, post-buckling equilibrium paths, and damage-driven stiffness loss within a single coupled formulation.*

Keywords
Composite shells; Nonlinear buckling; Progressive damage; CFRP; GFRP; Finite element modeling

1. Introduction

Composite cylindrical and conical shells are critical load-bearing components in aerospace structures such as fuselage sections, payload fairings, and launch vehicle casings. Compared to metallic shells, CFRP and GFRP laminates offer superior weight efficiency and tailored stiffness; however, their structural stability is significantly influenced by anisotropy, manufacturing imperfections, and damage accumulation.

Buckling often governs the design of thin-walled composite shells, yet experimental studies have demonstrated that material damage may initiate before or immediately after buckling, altering the post-buckling response and reducing residual strength. Classical shell theories and linear eigenvalue buckling analyses fail to capture this interaction, motivating the development of coupled nonlinear buckling–damage models.

The objective is to establish a mathematically consistent model that couples geometric nonlinearity and progressive damage, providing a foundation for high-fidelity finite element simulations and subsequent numerical validation.

2. Nonlinear Shell Kinematics

The shell kinematics are formulated using von Kármán nonlinear strain–displacement relations, suitable for thin-walled structures undergoing moderate rotations and large transverse displacements.

The total strain vector is decomposed as

$$\varepsilon = \varepsilon^{L} + \varepsilon^{NL}$$

where ε^{L} represents linear membrane and bending strains, and ε^{NL} accounts for geometric nonlinear terms arising from transverse displacement gradients.

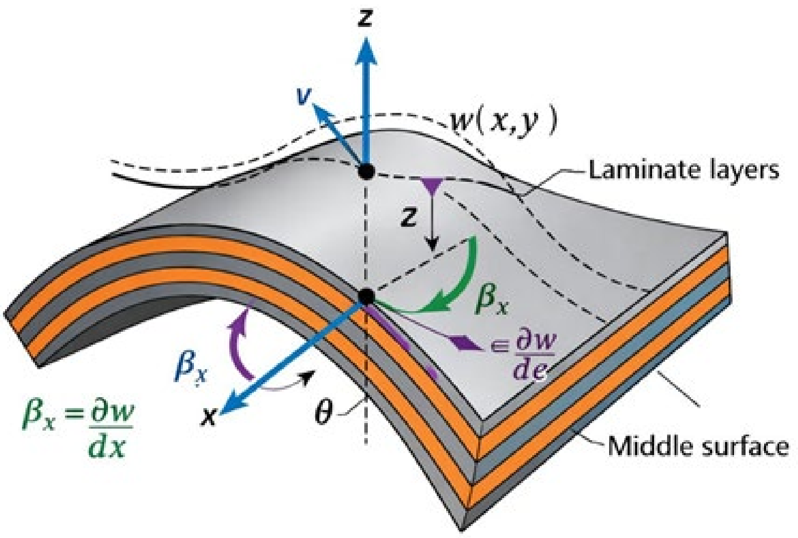

Figure 1 - Shell coordinate system and nonlinear kinematic variables for a laminated composite cylindrical shell.

This formulation captures the coupling between in-plane stresses and out-of-plane deformations that governs both pre-buckling and post-buckling behavior. The nonlinear kinematic relations form the basis for stability analysis and post-buckling path tracing.

3. Constitutive Modeling of Laminated Composite Shells

Each lamina is modeled as an orthotropic elastic material under plane-stress conditions. The constitutive relationship is expressed as

$$\sigma = C\_d \, \varepsilon$$

where C_d is the damage-dependent stiffness matrix. In the undamaged state, the stiffness matrix is obtained using classical lamination theory, accounting for ply orientation and stacking sequence.

The laminate extensional, coupling, and bending stiffness matrices (A, B, D) are computed by through-thickness integration of lamina properties. Symmetric and unsymmetric laminates can be accommodated, allowing investigation of bending–stretching coupling effects on buckling behavior.

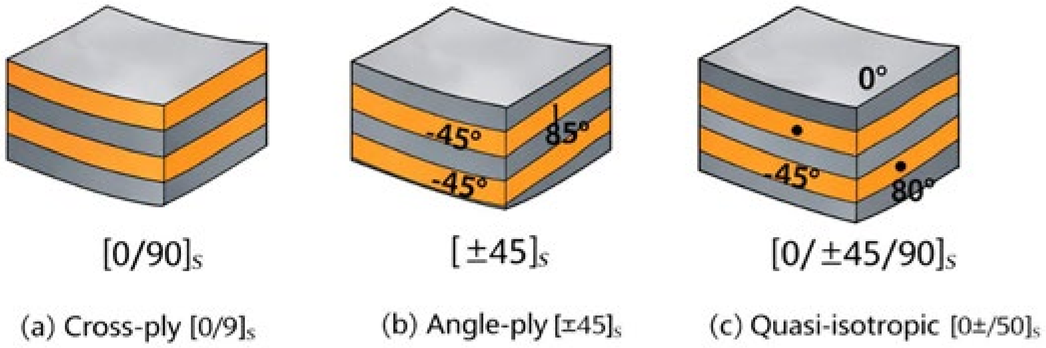

Figure 2 - Typical laminate stacking sequences used in composite shell analysis: ***(a) cross-ply [0/90]s, (b) angle-ply [±45]s, and (c) quasi-isotropic [0/±45/90]s.***

4. Damage Initiation Criteria

Intralaminar damage initiation is evaluated using ply-level failure criteria that distinguish between fiber-dominated and matrix-dominated failure modes. Separate indices are defined for fiber tension, fiber compression, matrix tension, and matrix compression.Damage initiation is assumed to occur when a failure index reaches unity:

$$F\_i = 1$$

This multi-mode approach enables accurate identification of the dominant failure mechanisms associated with different loading and deformation states, particularly in post-buckling regimes where stress redistribution occurs.

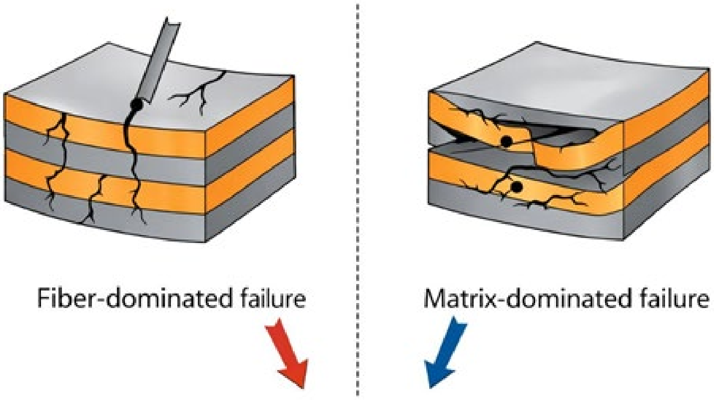

Figure 3 - Schematic representation of fiber- and matrix-dominated failure modes in laminated composites.

5. Damage Evolution and Stiffness Degradation

Following damage initiation, progressive degradation of material stiffness is modeled using continuum damage mechanics. Scalar damage variables are introduced to reduce the elastic properties of the lamina:

$$C\_d = (1 - d)\ C\_0$$

where C_0 is the undamaged stiffness matrix and d is a damage variable evolving with increasing strain or energy release. This formulation ensures a gradual stiffness reduction, allowing stable numerical simulation of post-buckling behavior and damage growth.

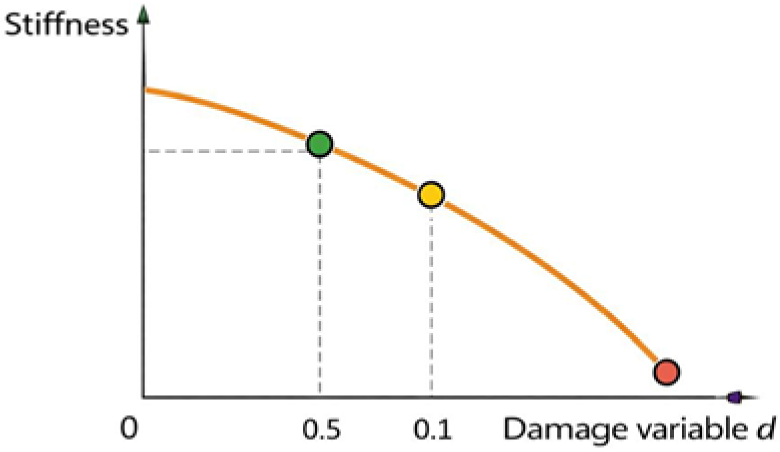

Figure 4 - Conceptual evolution of stiffness degradation as a function of damage variable d.

6. Coupled Buckling–Damage Formulation

Buckling and damage are solved simultaneously through nonlinear equilibrium equations with damage-dependent stiffness.

The nonlinear equilibrium of the composite shell is governed by

$$K(u, d)\, u = F$$

where K is the tangent stiffness matrix dependent on both displacement u and damage state d. Buckling arises naturally as a stiffness degradation phenomenon driven by geometric nonlinearity and further accelerated by damage evolution. This fully coupled formulation allows simultaneous prediction of:

Critical buckling loads, Post-buckling equilibrium paths , Damage localization and propagation, Ultimate failure loads

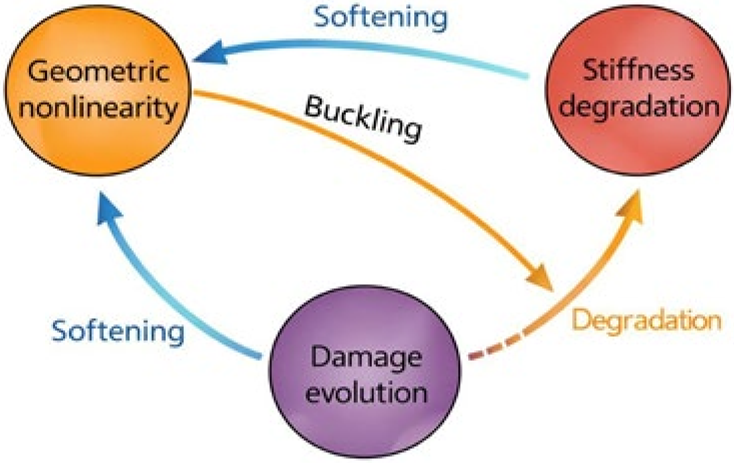

Figure 5 - Coupled interaction between geometric nonlinearity, damage evolution, and stiffness degradation.

7. Numerical Implementation

The proposed formulation is implemented within a nonlinear finite element framework using shell elements with through-thickness integration. Progressive damage is incorporated via user-defined material subroutines, enabling stiffness degradation at the ply level. Nonlinear solution strategies, including arc-length methods, are employed to trace post-buckling equilibrium paths.

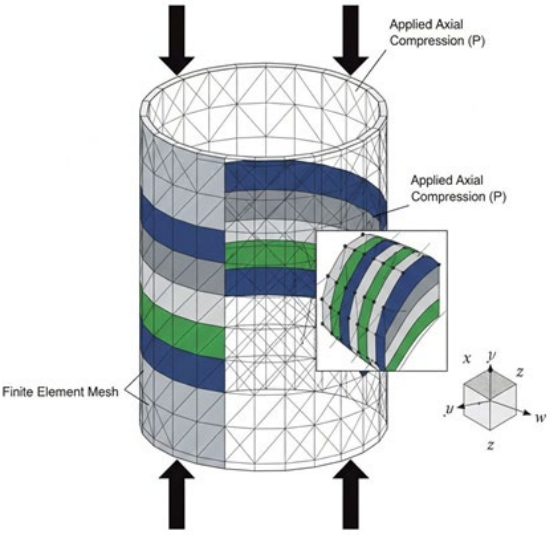

Figure 6 - Finite element model of a laminated composite cylindrical shell with applied axial compression.

8. Results and Discussion

Placeholders for buckling response, damage evolution, laminate comparison, and validation results.

8.1 Nonlinear Buckling and Post-Buckling Response

The load–displacement responses of CFRP and GFRP cylindrical shells exhibit a pronounced nonlinear behavior prior to classical buckling, confirming the strong influence of geometric nonlinearity. In contrast to linear eigenvalue predictions, the present model captures a gradual stiffness reduction leading to buckling, rather than a sharp bifurcation point. This behavior is particularly evident in CFRP shells, where higher anisotropy results in earlier nonlinear coupling between axial compression and transverse displacements.

Post-buckling equilibrium paths demonstrate stable and unstable branches depending on laminate configuration. Shells with quasi-isotropic layups exhibit higher critical loads and smoother post-buckling responses, while cross-ply and angle-ply laminates show increased imperfection sensitivity and more pronounced stiffness degradation after buckling. These trends are consistent with classical shell stability theory and previously reported experimental observations.

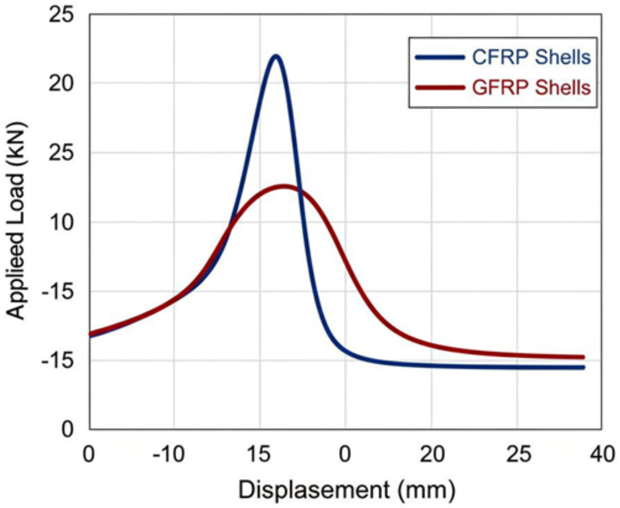

Figure 7 - Nonlinear load–displacement response of CFRP and GFRP composite shells.

8.2 Progressive Damage Initiation and Evolution

Damage initiation is observed to occur in the vicinity of local buckling waves, where stress concentrations develop due to nonlinear deformation. For CFRP laminates, fiber-dominated damage modes are prevalent under axial compression, particularly in 0° plies aligned with the loading direction. In contrast, GFRP shells exhibit earlier matrix-dominated damage due to their lower matrix stiffness and strength.

As loading progresses into the post-buckling regime, damage evolves progressively and spreads along the shell circumference and axial direction. The coupled formulation reveals that damage growth accelerates stiffness degradation, which in turn amplifies geometric nonlinearity, creating a strong feedback mechanism. This interaction significantly alters the post-buckling path and reduces the ultimate load-carrying capacity compared to undamaged predictions.

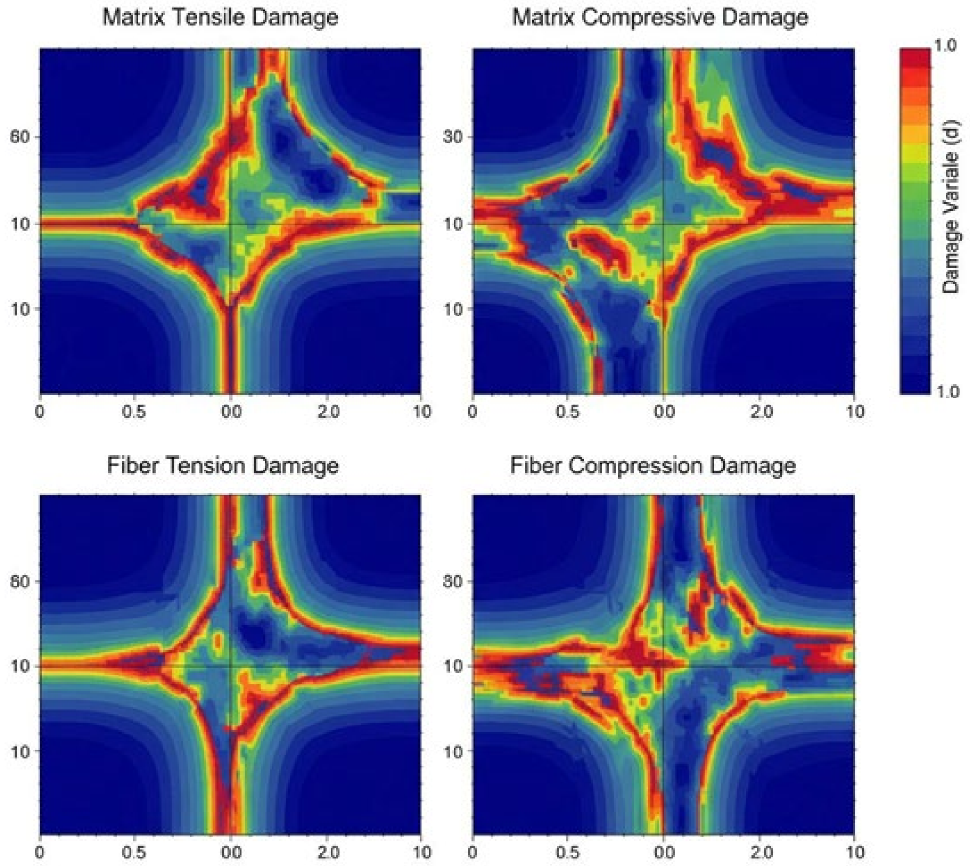

Figure 8 - Contour plots of damage variables during post-buckling deformation.

8.3 Effect of Laminate Configuration and Material System

The influence of laminate stacking sequence is found to be substantial. Symmetric quasi-isotropic laminates provide the highest buckling resistance and delayed damage initiation, owing to their balanced in-plane stiffness and reduced bending–stretching coupling. Angle-ply laminates, while beneficial for shear-dominated applications, show lower buckling loads and increased susceptibility to matrix cracking in the post-buckling regime.

Comparisons between CFRP and GFRP shells highlight the role of material stiffness and strength. CFRP shells sustain higher critical and ultimate loads but exhibit more localized damage once fiber failure initiates. GFRP shells, although less stiff, demonstrate more distributed damage patterns, leading to a more gradual loss of load-carrying capacity. These results underscore the importance of material selection and laminate tailoring in stability-driven design.

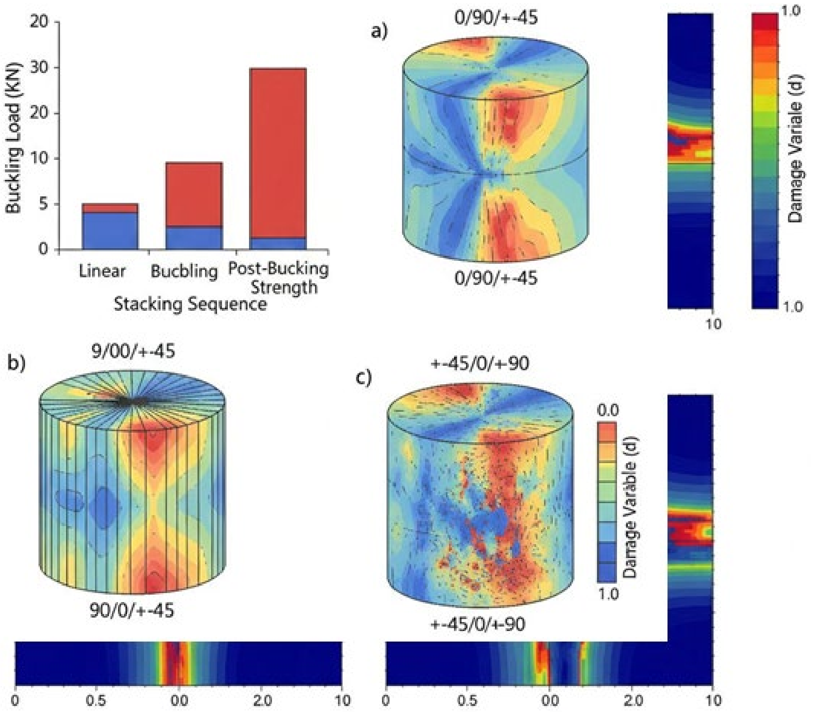

Figure 9 - Comparison of buckling loads and damage patterns for different stacking sequences.

8.4 Model Validation and Physical Interpretation

The predicted buckling loads and post-buckling responses show good qualitative agreement with experimental benchmarks reported in the literature. In particular, the model successfully reproduces the experimentally observed reduction in buckling strength due to damage initiation near the critical load. The ability of the coupled framework to capture both instability-driven deformation patterns and damage localization represents a significant improvement over uncoupled or purely elastic buckling analyses.

Overall, the results confirm that neglecting progressive damage can lead to non-conservative predictions of post-buckling strength and failure modes. The coupled nonlinear buckling–damage formulation provides a physically consistent explanation for the observed degradation of stiffness and strength in composite shells subjected to compressive loading, reinforcing the necessity of integrated stability–damage modeling for advanced composite structures.

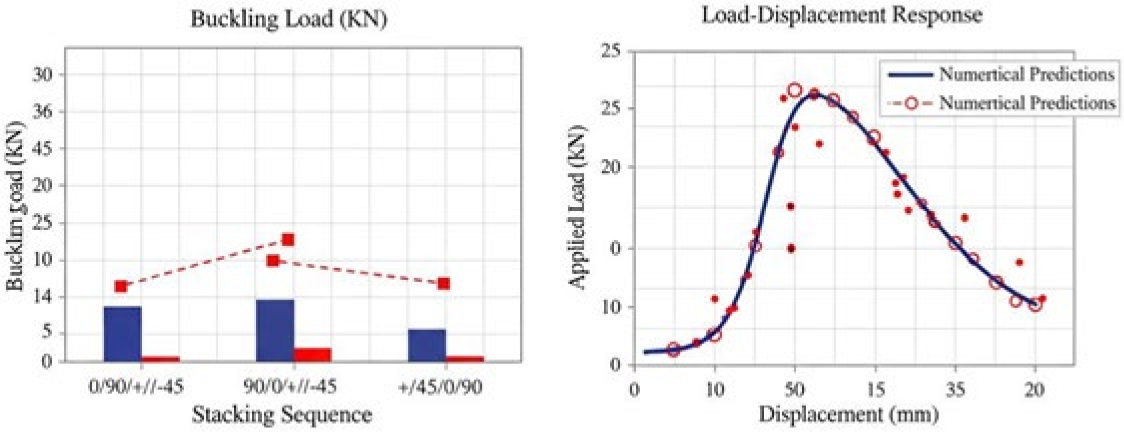

Figure 10 - Comparison between numerical predictions and experimental benchmark data.

9. Conclusions

This paper presents a coupled nonlinear buckling and progressive damage framework for laminated composite shells. By integrating nonlinear shell kinematics, anisotropic constitutive behavior, and damage evolution laws, the model captures the essential interaction between instability and material degradation. The formulation provides a rigorous theoretical basis for finite element implementation and enables accurate prediction of buckling, post-buckling response, and failure in CFRP and GFRP shells. Future work will extend the framework to combined loading conditions and environmental effects.

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