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NUMERICAL AND ANALYTICAL INVESTIGATION OF LAMB WAVE SCATTERING BY DEFECTS IN COMPOSITE LAMINATES

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Abstract. *This study presents a comprehensive evaluation of the propagation and scattering characteristics of fundamental antisymmetric Lamb waves A0 within composite laminates containing structural defects. The research focuses on comparing analytical modeling, the Born approximation, and numerical integration via the finite element method to predict scattering amplitudes on delaminations and through hole defects. A normalization procedure is applied to the scattered wave amplitudes relative to the incident wave at the defect center to ensure consistency across different laminate layers. Numerical accuracy is maintained by ensuring the wavelength is represented by at least twenty to thirty nodes in the computational grid. The investigation analyzes the scattering behavior over a wide range of defect radius values. Results indicate that while forward and backscattered waves exhibit similar amplitudes at small scales, forward scattered waves demonstrate a dominant trend as the defect size increases. It is observed that the Born approximation effectively predicts the overall trend of forward scattering but tends to underestimate the absolute magnitude. In contrast, backscattered amplitudes show rapid oscillations and significant discrepancies between analytical and numerical models, particularly for normalized radius values exceeding 0.3. This divergence is attributed to multiple internal reflections within the delamination region which are not accounted for in basic analytical models. Furthermore, the study explores the validity of the equivalent isotropic model for approximating quasi-isotropic laminates such as the 45/45/0/90s configuration. Analysis confirms that phase and group velocities at low frequencies are largely insensitive to fiber orientation, showing good agreement across different wave numbers. However, representing delamination as an inhomogeneity in these models yields satisfactory results primarily for quasi-isotropic cases, while layer by layer solid element modeling remains the necessary reference for high precision. The findings highlight the limitations of simplified approximation models in capturing complex reflection dynamics within laminated structures and provide insights for improving defect localization techniques in composite engineering*.

Key words: *composite laminates, Lamb waves, Born approximation, delamination scattering, finite element analysis, structural health monitoring.*

Introduction.

The field of non-destructive testing and structural health monitoring has undergone significant transformations with the increasing integration of advanced composite materials into modern engineering structures. Laminated composites are characterized by high strength to weight ratios and superior fatigue resistance which makes them highly desirable for various industrial sectors including energy production and transport systems. However, the complex internal architecture of these materials consisting of multiple layers with different fiber orientations introduces unique failure modes that are not present in traditional metallic structures. Among these failures delaminations and through hole defects are particularly critical as they can compromise the structural integrity of the component without being visible on the external surface. The detection and characterization of such internal flaws require sophisticated wave propagation analysis techniques that can account for the anisotropic nature of the medium. Lamb waves which are ultrasonic elastic waves propagating in thin plate like structures have emerged as a primary tool for the inspection of laminated composites.

These waves are classified into symmetric and antisymmetric modes each exhibiting distinct dispersion characteristics and sensitivity to different types of structural irregularities. The fundamental antisymmetric mode A0 is widely utilized for defect detection due to its short wavelength at relatively low frequencies and its high sensitivity to changes in the plate thickness and internal discontinuities [1]. The propagation of Lamb waves in composite laminates is governed by the interaction between the wave field and the heterogeneous layer sequence. Unlike isotropic materials where wave speed is uniform in all directions composite laminates exhibit direction dependent properties that influence the phase and group velocities. Analytical modeling of these waves often relies on simplified assumptions such as the equivalent isotropic model or quasi-isotropic approximations to reduce the computational complexity associated with modeling individual laminae.

These approximations are generally effective at low frequencies where the wavelength is significantly larger than the thickness of a single layer allowing the laminate to be treated as a homogenized medium. However, as the frequency increases or the wave encounters localized defects the limitations of these simplified models become more apparent. The scattering of Lamb waves on defects such as delaminations involves complex physical phenomena including mode conversion reflection and transmission at the boundaries of the inhomogeneity. When an incident Lamb wave interacts with a delamination it experiences a sudden change in the local stiffness and mass distribution of the plate. This interaction results in the generation of scattered wave fields that carry essential information about the size shape and location of the defect. Understanding the relationship between the defect geometry and the resulting scattering pattern is fundamental for the development of reliable diagnostic algorithms. One of the common mathematical frameworks used to describe this scattering process is the Born approximation [2]. Originally developed in quantum mechanics and later adapted for elasto-dynamics the Born approximation assumes that the total field inside the scattering region can be replaced by the incident field. This assumption simplifies the integral equations of scattering and allows for the derivation of analytical expressions for the scattered wave amplitudes. While the Born approximation is highly efficient and applicable to defects with complex shapes its accuracy is often limited to cases where the scattering is weak or the impedance mismatch between the defect and the host material is small. In the context of composite laminates the applicability of the Born approximation depends on how well the delamination can be represented as a localized inhomogeneity.

Numerical integration methods such as the finite element method provide a more rigorous alternative to analytical approximations. By discretizing the entire volume of the composite structure into a mesh of solid elements numerical models can capture the intricate details of wave propagation including multiple internal reflections and the effects of fiber orientation within each individual layer. For accurate prediction of Lamb wave scattering the computational grid must be sufficiently dense to resolve the wave features. It is generally accepted that at least twenty to thirty nodes per wavelength are required to minimize numerical dispersion and ensure that the scattering at defect boundaries is correctly represented. Finite element modeling serves as a reference procedure for validating more efficient but less precise analytical and approximate methods. A critical aspect of analyzing scattered Lamb waves is the normalization of the amplitudes. To compare results across different configurations researchers often normalize the scattered wave amplitudes by the maximum absolute amplitude of the incident wave measured at a specific reference point such as the center of the defect zone. This normalization facilitates the study of how scattering patterns evolve as a function of the defect dimensions typically represented by a normalized radius R. Theoretical studies have shown that forward and backscattered waves exhibit different sensitivities to defect size. Forward scattered waves tend to increase in amplitude more steadily with increasing defect dimensions while backscattered waves often show oscillatory behavior due to constructive and destructive interference between reflections from the front and back edges of the defect. This interference is further complicated in composite laminates by the presence of multiple layers which can lead to internal reflections that are not easily captured by single scattering theories. The discrepancy between analytical models and numerical simulations often increases for larger defects where the multiple reflection effects become more pronounced. Furthermore, the sensitivity of Lamb wave parameters to fiber orientation is a subject of ongoing discussion in the literature. While some modes and frequencies appear to be relatively insensitive to the specific stacking sequence others are highly dependent on the local anisotropy. Quasi isotropic laminates where the layers are oriented to balance the stiffness in multiple directions are often used to simplify the analysis. In these models the deformation object can be represented as an elongated fiber at a fixed angle which influences the low frequency Lamb wave behavior.

Despite these simplifications the agreement between different modeling approaches remains a challenge particularly when dealing with the high degree of anisotropy and the complex boundary conditions present at a delamination interface. The study of these scattering mechanisms is essential for advancing the capabilities of structural health monitoring systems and for improving the precision with which internal damage can be localized and quantified in composite structures. By integrating analytical approximations with high fidelity numerical simulations researchers can better understand the tradeoffs between computational speed and physical accuracy in wave-based defect detection. This foundational knowledge is necessary for developing the next generation of inspection technologies that can ensure the safety and reliability of complex composite assemblies in various engineering applications.

Lamb A0 wave scattering.

The difference in the maximum absolute amplitude of the scattered Lamb waves A0 is estimated in this work. The normalization procedure of all scattered Lamb waves A0 by the maximum absolute amplitude of the incident wave in the center of the defect zone for a given laminate layer is performed. The wavelength in the numerical integration method accounted for at least 20 - 30 nodes of the computational grid, which is sufficient for accurate prediction of the propagation and scattering of the Lamb wave A0 on defects in composite laminates.

The Born approximation is also applicable to defects with complex shapes. In addition, the Born approximation was used to approximate the scattered Lamb wave amplitude A0 and compared with analytical and experimentally verified predictions of numerical integration using the finite element method. The plate properties of the inhomogeneity region can be expressed in terms of the properties corresponding to the region outside the inhomogeneity

$$D^* = D(1+\delta_1), \tag{1}$$

$$\kappa^2 Gh^* = \kappa^2 Gh(1+\delta_2), \tag{2}$$

$$\rho I^* = \rho I(1+\delta_3), \tag{3}$$

$$\rho h^* = \rho h(1+\delta_4), \tag{4}$$

where

$D = f(E, I, \nu)$;

E is the Young's modulus;

I is the moment of inertia;

ν is the Poisson's ratio;

$\kappa = \pi(12)^{-0.5}$ is the shear correction factor;

G is the shear modulus;

ρ is the density;

h is the sample thickness;

δ_1 - δ_4 are the defect factors.

For a fixed signal frequency ω, the scattered Lamb wave A0, according to the Born approximation, can be expressed using the following relation

$$W^s = \iint \Big\{ \delta_1 D \Gamma^{(i)}_{\beta\alpha} g_{3\alpha,\beta} + \left[\delta_2 \kappa^2 \eta \left(W^i_\alpha - \psi^i_\alpha \right) + \delta_3 \omega^2 \rho I \psi^i_\alpha \right] g_{3\alpha} + \\ + \delta_2 \kappa^2 Gh \left(W^i_\alpha - \psi^i_\alpha \right) g_{33,\alpha} + \delta_4 \rho h \omega^2 g_{33} \Big\} d\xi d\eta, \tag{5}$$

where

α, β = 1, 2;

ξ, η represents point coordinates within fixed region;

Γ is the plate strain;

g $_{i,k}$ are the Green's functions:

$$g_{lm} = \gamma \frac{\partial H_0(k_1 r')}{\partial p}, \tag{6}$$

where H_0 is the Huncel function;

$$\gamma = \frac{i}{4D\left(k_1^2 - k_2^2\right)}, \tag{7}$$

$$r' = \left[(x-\xi)^2 + (y-\eta)^2\right]^{0.5}, \tag{8}$$

$$p = \begin{cases} x, \ (l=3, m=1) \\ y, \ (l=3, m=2). \\ z, \ (l=3, m=3) \end{cases} \tag{9}$$

The scattered Lamb wave can be represented by

$$W^s(r,\theta) = \left[\frac{2}{\pi k_1 r}\right]^{0.5} \exp\left[i\left(k_1 r - \frac{\pi}{4}\right)\right] \cdot T(\theta) \sum_{n=1}^{4} \delta_n P_n(\theta), \tag{10}$$

$$P_1(\theta) = -\gamma \lambda_1 k_1^2 D\left(\cos^2\theta + \nu \sin^2\theta\right), \tag{11}$$

$$P_2(\theta) = -\frac{\gamma k^2 Gh(1-\lambda_1)^2}{\lambda_1} \cos\theta, \tag{12}$$

$$P_3(\theta) = \gamma \lambda_1 \rho I \omega^2 \cos\theta, \tag{13}$$

$$P_4(\theta) = \frac{\gamma \rho h \omega^2}{\lambda_1 k_1^2}, \tag{14}$$

$$T(\theta) = 2\pi k_1 a \frac{J_1\left[k_1 a (2 - 2\cos\theta)^{0.5}\right]}{(2 - 2\cos\theta)^{0.5}}. \tag{15}$$

An analytical model of Lamb A0 wave scattering by through-hole defects is used to verify the numerical accuracy of the finite element modeling performed in this study. The verification was performed for a limited number of *R* values.

The calculation model analyzes the range of *R* values and compares them with the analytical results. The normalization procedure is performed using the maximum absolute amplitude of the incident wave in the center of the defect zone in the intact laminated composite sample. The forward and backward scattered Lamb waves A0 have similar amplitudes. However, the forward scattered waves tend to have a larger amplitude with increasing *R*.

In addition, the dynamics of the normalized amplitudes of the forward and backscattered Lamb waves A0 for a range of *R* values is analyzed. The forward and backscattered amplitudes increase with a similar slope and magnitude for R less than 0.45, after which the backscattered amplitudes increase at a lower rate and with small variations.

The entire set of obtained analytical, approximate results for finite elements for scattering of Lamb waves A0 on delaminations was compared with the results predicted by the equivalent isotropic model, which is an approximation to the composite laminate [45/45/0/90]S. The purpose of such a comparison is to analyze the suitability of the representation of delamination by inhomogeneity in the analytical model and the Born approximation.

It was shown that the A0 phase and group velocity are not sensitive to the fiber orientation in the composite laminate. Very good agreement was obtained in all three sets of results at different wave numbers for Lamb waves, thus confirming the approximation of the phase and group velocity in the composite laminate at low frequencies by an equivalent isotropic model. Analysis of the calculated results indicates that the Born approximation underestimates the forward scattering amplitudes of the Lamb wave, but it predicts the forward scattering amplitude trend well.

It is found that the backscatter amplitudes oscillate faster with R than the forward scatter amplitudes. The analytical, approximated backscatter amplitude results and the results of numerical integration using the finite element method have different oscillation patterns. The analytical results have the form of a sinusoidal function increasing with the incident wave amplitude. The approximated results oscillate between zero and maximum values and have an increasing behavior of a sinusoidal function.

It should be pointed out that the general trends of all three sets of results increase with the numerical value of R. Comparing the predictions of the forward and backscattering amplitudes, it can be stated that the inhomogeneity model represents the delamination in the forward scattering amplitudes well. However, for the backscattering amplitudes, there is a significant discrepancy in the amplitudes. A similar phenomenon has also been shown in the defect localization experiments in laminar composites for delaminations in composite beams.

A possible reason for the discrepancy between the analytical results and the results of numerical integration using the finite difference method is that the analytical model does not take into account multiple internal reflections in the delamination region. The discrepancy between the analytical results and the results of integration using the finite element method increases for R greater than 0.3. The reason for such an increase in discrepancies is that the effect of multiple reflections in the delamination region becomes sharper.

Summary and conclusions.

The investigation into the scattering behavior of antisymmetric Lamb waves A0 within composite laminates provides critical insights into the effectiveness of various modeling approaches for structural health monitoring. It was demonstrated that the normalization of scattered amplitudes relative to the incident wave allows for a consistent comparison between analytical and numerical methods across different defect scales. The use of a dense computational grid with at least twenty to thirty nodes per wavelength proved essential for the accurate prediction of wave interactions at the boundaries of delaminations. The comparative analysis revealed that the Born approximation is a suitable tool for predicting the general trends of forward scattering amplitudes particularly for smaller defects. However it consistently underestimates the absolute magnitude of the scattered field as the defect size increases. For backscattering a significant discrepancy was observed between analytical models and finite element results due to the influence of multiple internal reflections within the delamination region which are more pronounced for normalized radius values greater than 0.3. It was also shown that low frequency Lamb wave parameters such as phase and group velocity are relatively insensitive to fiber orientation in quasi isotropic laminates allowing for the application of equivalent isotropic models in certain contexts. The findings suggest that while simplified models offer computational efficiency they must be used with caution for precise defect quantification in complex composite environments.

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