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EQUATION OF MOTION OF FLOWS WITH SUSPENDED PARTICLES

РІВНЯННЯ РУХУ ПОТОКІВ ІЗ ЗАВИСЛИМИ ЧАСТИНКАМИ

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Abstract. *Recently, it has been increasingly confirmed that the transport of suspended particles is a vital function for the regional and global carbon and nutrient cycle during their transport in surface waters. Usually they include fine sand, silt and clay, which can absorb carbon and nutrients. Numerous works of domestic scientific institutions are devoted to the development of refined recommendations for determining the granulometric composition of suspended sediments and the formation of a methodology for calculating spatial deformations of channels. In particular, this is achieved by applying mathematical and physical modeling methods. This work considers two approaches to describing transport processes in channel flows, and also establishes that the main and defining characteristics of the turbulent structure of the flow are its turbulent tangential stresses.*

Calculation methods were presented and the calculated distribution of velocities over the depth of the turbulent suspended flow was obtained. From it we can conclude that the transport phenomena in this case are characterized by more complex transport coefficients, which are related to viscosity and density.

Keywords: *transport processes, channel flows, equations of motion, suspended particles, velocity distribution.*

Introduction.

Recently, there is increasing evidence that the transport of suspended particles plays a vital role in the regional and global carbon and nutrient cycles by transferring them to rivers, lakes or oceans. They usually consist of fine sand, silt and clay, which can absorb carbon and nutrients [1].

Sedimentation processes are activated in areas of changing environmental conditions, where such characteristics as the nature of the underlying surface, the speed and direction of movement, the chemical composition of water, and the species composition of biota change. Since 1983, much work by domestic scientific institutions has been devoted to the development of refined recommendations for determining the granulometric composition of suspended sediments and the formation of a methodology for calculating spatial deformations of river channels. In particular, this is achieved by applying mathematical and physical modeling methods [2].

The main text

There are two approaches to describing transport processes in such environments. The first approach is associated with the formation of separate equations characterizing the liquid and solid phases. In this approach, the equations of motion of the flow of a liquid mixture are used for the environments [3–5]

$$\rho(1-S)\left(\partial \mho_i / \partial t + \mho_j \partial \mho_i / \partial x_j\right) = -(1-S)\partial P / \partial x_i + (1-S)\partial \tau_{ij} / \partial x_j + $$
$$+K\left(\mho_i - \mho_{is}\right) + F_i - (1-S)L\mho_i \quad (1)$$

In the second case, taking into account that the density of the mixture can be given by dependence (2), the equation for a continuous medium of motion of a flow of a mixture homogeneous in size takes the form

$$\rho S\left(\partial \mho_{is} / \partial t + \mho_{js} \partial \mho_{is} / \partial x_j\right) = -S\partial P / \partial x_i + S\partial \tau_{ijs} / \partial x_j + K\left(\mho_{is} - \mho_i\right) + F_{is} - SL_s\mho_{is}, \quad (2)$$

where $\mho_{is}, \mho_i$ – velocity of liquid and mixture particles, F_{is} – mass force, $\partial \tau_{ij}$ – stress tensor component, K – ratio of both phases, L – mass transfer coefficient between flow layers.

When considering the motion of each phase separately, external forces are distributed to each phase. In addition, there is an interaction force between the phases, which when considering the mixture as a whole is an internal force. It follows that the forces acting on each phase consist of surface and mass interaction forces. Given this, the equation of motion for the *i*-th phase in differential form can be written as [3–5]

$$\rho_1 d\overline{U_1} / dt = div\overline{P_i} + \overline{\Pi_i} + \rho_i \overline{F_i}. \quad (3)$$

Here $P_i = -f_i gradP + \partial \tau_{ij} / \partial x_{ij}$ stress tensor acting on the *i*-th phase; τ_{ij} – internal tension; f_i – volume content of the i-th phase (*i*=1,2).

The interaction force, depending on the difference in the speeds of each phase, is considered in the form

$$\Pi_i = K\left(\overline{U} - \overline{U_i}\right) \quad (4)$$

Substituting the values of the parameters, equation (1) can be written as

$$\rho_i du_i / dt = -f_i gradP + \partial\tau_{ij} / \partial x_{ij} + K\left(\overline{U} - \overline{U_s}\right) + \rho_i l\,, \tag{5}$$

where $\overline{U}$, $\overline{U_s}$ – corresponding velocity vectors of the carrier fluid and solid particle; K – interaction coefficient between the carrier liquid and solid particle phases; F_i – mass forces, in this case, mainly the components of gravity are considered.

The main and defining characteristics of the turbulent flow structure are turbulent tangential stresses. Turbulent tangential stresses are determined according to the model developed in the work [3–4]:

$$\tau_{ijl} = -f_i\mu_i(-\frac{2}{3}div\overline{V_k} + 2\frac{\partial\upsilon_{kj}}{\partial x_j} + \int LV_i dn \text{ when } j = l\left(\overline{n} = 1.2\right)$$

$$\tau_{ijl} = -f_i\mu_i\left(\frac{\partial\upsilon_{kl}}{\partial x_j} + \frac{\partial\upsilon_{kj}}{\partial x_j}\right) \text{ when } \mathrm{j} \neq l\,. \tag{6}$$

Then equation (5), taking into account (6), in the projection on the coordinate axis *(j=x, y, z; l= x, y, z)* for the first phase of the flow with suspended particles we write in the following form [3–4].

$$\begin{cases} \rho_1\frac{du_1}{dt} = \frac{\partial}{\partial x}\left(2f_1\mu_1\frac{\partial u_1}{\partial x}\right) + \frac{\partial}{\partial y}\left(2f_1\mu_1\left(\frac{\partial u_1}{\partial y} + \frac{\partial\upsilon_1}{\partial x}\right)\right) + \\ + \frac{\partial}{\partial z}\left(2f_1\mu_1\left(\frac{du_1}{dz} + \frac{d\omega_1}{dx}\right)\right) - L_1u_1 + K\left(u_1 - u_2\right)\rho_1F_x \\ \rho_1\frac{d\upsilon_1}{dt} = \frac{\partial}{\partial y}\left(2f_1\mu_1\frac{\partial\upsilon_1}{\partial y}\right) + \frac{\partial}{\partial x}\left(2f_1\mu_1\left(\frac{\partial u_1}{\partial x} + \frac{\partial\upsilon_1}{\partial y}\right)\right) + \\ + \frac{\partial}{\partial z}\left(2f_1\mu_1\left(\frac{du_1}{dz} + \frac{d\omega_1}{dy}\right)\right) - L_1\upsilon_1 + K\left(u_1 - u_2\right)\rho_1F_y \\ \rho_1\frac{d\omega_1}{dt} = \frac{\partial}{\partial z}\left(2f_1\mu_1\frac{\partial\omega_1}{\partial z}\right) + \frac{\partial}{\partial x}\left(2f_1\mu_1\left(\frac{\partial\omega_1}{\partial x} + \frac{\partial u_1}{\partial z}\right)\right) + \\ + \frac{\partial}{\partial y}\left(2f_1\mu_1\left(\frac{d\upsilon_1}{dz} + \frac{d\omega_1}{dy}\right)\right) - L_1\omega_1 + K\left(\omega_1 - \omega_2\right)\rho_1F_z \end{cases} \tag{7}$$

For the second phase, we write accordingly [3–4]

$$\begin{cases} \rho_2 \dfrac{du_2}{dt} = \dfrac{\partial}{\partial x}\left(2 f_2 \mu_2 \dfrac{\partial u_2}{\partial x}\right) + \dfrac{\partial}{\partial y}\left(2 f_2 \mu_2 \left(\dfrac{\partial u_2}{\partial y} + \dfrac{\partial \upsilon_2}{\partial x}\right)\right) + \\ + \dfrac{\partial}{\partial z}\left(2 f_2 \mu_2 \left(\dfrac{du_2}{dz} + \dfrac{d\omega_2}{dx}\right)\right) - L_1 u_1 + K\left(u_1 - u_2\right)\rho_1 F_x \\ \rho_2 \dfrac{d\upsilon_2}{dt} = \dfrac{\partial}{\partial y}\left(2 f_2 \mu_2 \dfrac{\partial \upsilon_2}{\partial y}\right) + \dfrac{\partial}{\partial x}\left(2 f_2 \mu_2 \left(\dfrac{\partial u_2}{\partial x} + \dfrac{\partial \upsilon_2}{\partial y}\right)\right) + \\ + \dfrac{\partial}{\partial z}\left(2 f_2 \mu_2 \left(\dfrac{du_2}{dz} + \dfrac{d\omega_2}{dy}\right)\right) - L_1 \upsilon_1 + K\left(u_1 - u_2\right)\rho_1 F_y \\ \rho_2 \dfrac{d\omega_2}{dt} = \dfrac{\partial}{\partial z}\left(2 f_2 \mu_2 \dfrac{\partial \omega_2}{\partial z}\right) + \dfrac{\partial}{\partial x}\left(2 f_2 \mu_2 \left(\dfrac{\partial \omega_2}{\partial x} + \dfrac{\partial u_2}{\partial z}\right)\right) + \\ + \dfrac{\partial}{\partial y}\left(2 f_2 \mu_2 \left(\dfrac{d\upsilon_2}{dz} + \dfrac{d\omega_2}{dy}\right)\right) - L_2 \omega_2 + K\left(\omega_1 - \omega_2\right)\rho_2 F_z \end{cases} \quad (8)$$

where *u, v, w* – velocity components in projections on the coordinate axis.

A feature of the proposed system of differential equations is that the right-hand side of the equations contains two additional terms (L_1, L_2), which characterize the turbulence of a channel flow with suspended particles (L_1, L_2), which is determined based on the developments given in the works [4, 6, 7].

To solve practical problems of channel flow, it is necessary to develop a method for calculating the distribution of the average velocity of a suspended flow, which is applicable in wide ranges of changes in the conditions of motion of the flow parameters of the carrier fluid and solid particles [6–8]. To develop a method for calculating the distribution of the mediated velocity over the depth of a flow with suspended particles in open channels, a system of differential equations (7, 8) in the case of a steady ($du / dt = 0$), one-dimensional ($du / dx = du / dz = 0$) and uniform motion, assuming that $f = s$ – the concentration of the second phase in the flow for the carrier fluid and for the flow of suspended particles can be written in the following form

$$\begin{cases} \dfrac{\partial}{\partial y}\left[\left(1 - s\right)\mu\left(\dfrac{\partial u_1}{\partial y}\right)\right] K\left(u_1 - u_2\right) - \left(1 - s\right) L_1 u_1 = -\left(1 - s\right)\rho g i \\ \dfrac{\partial}{\partial y}\left[\mu s\left(\dfrac{\partial u_2}{\partial y}\right)\right] + K\left(u_1 - u_2\right) - s L_2 u_2 = -s \rho_T g i \end{cases}, \quad (9)$$

where ρ, ρ_T – density of carrier liquid and solid particles kg/m [3]; *i* – slope of the water surface.

The system of differential equations (9) is solved numerically using the finite difference method. The results of the calculations show that the value of the velocity, i.e. the difference in the velocities of the carrier fluid and the solid particle along the depth of the flow, changes. This is obvious, since during the movement of a suspended flow in open channels the distribution of the concentration of suspended particles along the depth of the flow has an exponential character. Hence, with increasing depth, the concentration of suspended particles increases and there is a need to take into account the fractional composition, which significantly affects the distribution of the relative velocity along the depth of the flow (Fig. 1).

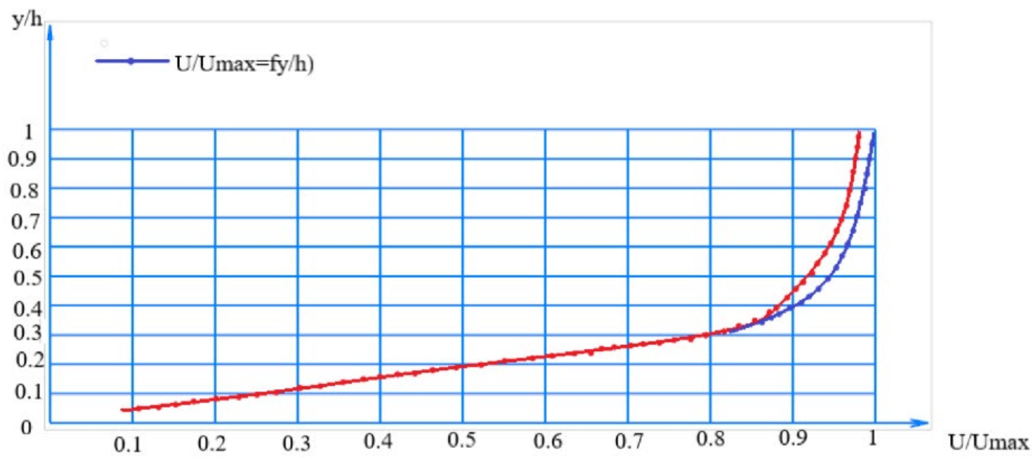

Fig. 1. Result of calculation of velocity distribution over the depth of turbulent flow with suspended particles [3–4]

Therefore, the transport phenomena in this case are characterized by more complex transport coefficients, which are related to viscosity and density.

Conclusion and findings. This paper considers two approaches to describing transport processes in channel flows, and also establishes that the main and defining characteristics of the turbulent structure of the flow are its turbulent tangential stresses. Methods for calculating the distribution of the average velocity of a flow with suspended particles, which are used in wide ranges of changes in the conditions of motion of the flow parameters of the carrier fluid and solid particles, were presented.

As a result of the calculation, using the finite difference method, the calculated distribution of velocities along the depth of a turbulent flow with suspended particles was obtained. From it, it can be concluded that the transport phenomena in this case are characterized by more complex transport coefficients, which are related to viscosity and density.

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Анотація. *Останнім часом все більше підтверджується, що транспортування завислих частинок є життєво важливою функцією для регіонального та глобального кругообігу вуглецю і поживних речовин під час їх переносу у поверхневих водах. Зазвичай до їх складу входить дрібний пісок, мул і глини, які можуть поглинати вуглець і поживні речовини. Численні роботи вітчизняних наукових установ присвячено розробленню уточнених рекомендацій для визначення гранулометричного складу завислих наносів та формуванню методики розрахунку просторових деформацій русел. Зокрема цього досягають застосовуючи методи математичного та фізичного моделювання. У даній роботі розглянуто два підходи опису процесів переносу у руслових потоків, а також встановлено, що головними та визначальними характеристиками турбулентної структури потоку є його турбулентні дотичні напруження.*

Було наведено методики розрахунку та одержано розрахунковий розподіл швидкостей по глибині турбулентного зваженого потоку. З нього можна зробити висновок, що явища переносу в цьому випадку характеризуються більш складними коефіцієнтами переносу, які пов'язані із в'язкістю та густиною.

Ключові слова: *процеси переносу, руслові потоки, рівняння руху, завислі частинки, розподіл швидкостей.*

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