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FINITE ELEMENT STUDY OF LAMB WAVE SCATTERING CHARACTERISTICS IN A QUASI-ISOTROPIC COMPOSITE LAMINATE

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Abstract. *This study presents an investigation into the propagation and scattering characteristics of Lamb waves in laminated composite structures, employing an alternative approach to the numerical Fourier transform for enhanced efficiency. The method models the motion of Lamb wave packets by analyzing their propagation along equivalent waveguides whose geometry relates directly to the composite sample dimensions. The finite element method is utilized to determine fundamental wave characteristics, including the phase and group velocities of the propagating wave packets. A critical aspect of this work involves the comprehensive verification of the developed finite element method model across a wide range of values for the ratio of the defect diameter to the incident wavelength. This extensive verification ensures the model's accuracy and robustness for practical applications in non-destructive evaluation. The research focuses specifically on the scattering phenomenon caused by delamination defects, which are modeled as discontinuities within the laminate. A key finding confirms that the presence of such a discontinuity induces both reflected and transmitted waves from the volumetric defect boundary, indicating a complex energy partitioning process. Furthermore, the analysis of scattered waves for the fundamental antisymmetric mode showed that this mode possesses increased sensitivity to small defects, emphasizing its importance for early-stage damage detection. To ensure consistent and comparable results, a crucial step in the post-processing procedure was the normalization of all scattered A0 Lamb waves. This was achieved by dividing the scattered wave amplitudes by the maximum absolute amplitude of the incident wave at the center of the defect zone for the specified laminate layer. In summary, this work establishes verified finite element method framework for accurately predicting Lamb wave propagation and scattering from delaminations in laminated composites.*

Key words: *Lamb wave, finite element method, laminated composite, delamination, scattering, waveguide.*

Introduction.

The increasing reliance on laminated composite materials across critical industries, including aerospace, automotive, and wind energy, is driven by their exceptional strength-to-weight ratio, high stiffness, and corrosion resistance. These advanced materials, often consisting of layers of fiber-reinforced polymer plies stacked at various orientations, offer significant advantages over traditional metallic structures, primarily in terms of weight reduction and tailored mechanical properties. However, these benefits come with a vulnerability to various internal defects, such as delaminations (interlaminar cracks), matrix cracks, and fiber fractures, which often initiate during manufacturing processes, resulting from imperfect curing or accidental impact damage, or emerge under service loading due to fatigue or environmental stress [1]. The presence of these flaws can severely compromise the structural integrity, load-bearing capacity, and residual life of the composite component, necessitating reliable and non-destructive methods for their early detection and accurate characterization. The inherent complexity of composite failure mechanisms, where damage often remains subsurface and invisible to visual inspection, further amplifies the need for advanced evaluation techniques.

Non-Destructive Evaluation (NDE) techniques, particularly those based on the use of guided waves, have emerged as a powerful and highly effective tool for inspecting large areas of thin-walled structures like composite laminates. Guided waves, such as Lamb waves, are especially suited for plate-like structures because they propagate along the entire thickness of the structure and can travel long distances with relatively low attenuation, enabling rapid screening of vast structural areas from a single or limited number of inspection points [2]. These waves interact sensitively with geometric boundaries and internal discontinuities. Lamb waves are characterized by their inherent dispersive nature, meaning their velocity depends not only on the material properties but also on the excitation frequency and the product of frequency and plate thickness, giving rise to an infinite number of propagating modes: symmetric and antisymmetric. Among these, the fundamental modes, S0 and A0, are of primary interest in NDE, as they are the only modes propagating at very low frequencies. Understanding and exploiting the propagation and scattering behavior of specific Lamb wave modes as they interact with defects is fundamental to developing effective NDE methodologies that can reliably distinguish between different damage types and accurately quantify their size and location.

The challenge in inspecting composite laminates lies not only in the damage morphology itself but also in the material's inherent anisotropy and complex lay-up structures, which significantly complicate wave mechanics compared to simpler isotropic materials. The mechanical properties of a laminate vary as a function of direction and position through the thickness. Despite their macroscopic isotropic appearance, these laminates still present considerable challenges for wave modeling due to their through-thickness heterogeneity, where each ply retains its strong directional anisotropy. The interaction of Lamb waves with critical subsurface defects, such as a localized delamination, is a complex wave-structure interaction phenomenon. A delamination acts as a local discontinuity or a structural break, where the single laminate effectively separates into two sub-laminates (upper and lower), locally altering the stiffness profile and fundamentally influencing wave reflection, transmission, and complex mode conversion (where an incident mode scatters into multiple other propagating modes). Accurately predicting these scattering characteristics and understanding the partition of energy among the scattered modes is crucial for reliable defect sizing and location.

Numerical simulation methods, particularly the Finite Element Method (FEM), play an indispensable and increasingly important role in advancing the understanding of Lamb wave mechanics and scattering phenomena in complex composite structures. Analytical solutions for wave propagation are typically limited to idealized, infinitely large, homogeneous, and isotropic plates, or to simple geometric defects where the boundary conditions are trivial. FEM, conversely, offers the capability to model the full complexity of material anisotropy, realistic three-dimensional geometry, and arbitrary defect shapes (like circular or elliptical delaminations) with high fidelity. FEM provides a virtual laboratory environment where controlled studies can be performed: one can isolate the scattering of a single incident Lamb wave mode, which is often difficult to achieve cleanly in physical experiments due to overlapping wave packets, environmental noise, and boundary reflections. Through high-resolution FEM simulations, the time-domain and frequency-domain wave fields can be visualized and analyzed in minute detail, leading to deeper insights into mode-conversion mechanisms, the distribution of scattered energy, and the near-field effects immediately adjacent to the defect.

A particular focus within NDE research is the potential of the fundamental antisymmetric mode (A0) for delamination detection. Research consistently indicates that the A0 mode often exhibits an increased sensitivity to small defects compared to the fundamental symmetric mode (S0) and the fundamental shear horizontal mode (SH0) at the same excitation frequency. This enhanced sensitivity is primarily attributed to the A0 mode's dominance of out-of-plane motion and bending strain, which are profoundly affected by a delamination that locally reduces the bending stiffness. This makes the A0 mode a prime candidate for high-resolution, early-stage damage detection. The methodology for quantifying scattering often involves analyzing the difference signal, which is the time-domain signal obtained by subtracting the signal from a pristine (undamaged) panel from the signal obtained from the damaged panel at the same monitoring point. This difference signal represents the scattered field and is used to quantify the defect's influence on the wave propagation path. Advanced analytical techniques, such as the Mindlin plate theory combined with the Born approximation, provide theoretical frameworks for simplifying the scattering analysis. While offering benefits in computational efficiency, these analytical models often rely on the concept of an equivalent isotropic model to simplify the complex anisotropic material behavior of the quasi-isotropic laminate.

Furthermore, a significant practical challenge in NDE is the influence of the defect size relative to the incident wavelength (R value). While a limited number of experimental studies have focused on specific, narrow ranges of R values, a comprehensive numerical verification of the FEM model across a wide range of R values is essential for validating the predictive capabilities of the model and ensuring its practical applicability across different inspection scenarios, operating frequencies, and defect sizes. This comprehensive approach is necessary to establish confidence in the numerical model's ability to accurately represent real-world defect detection scenarios. Additionally, to ensure consistent and comparable analysis across different excitation frequencies and defect sizes, a robust normalization procedure is required. Specifically, the scattered A0 Lamb wave amplitudes must be normalized against a reference value, such as the maximum absolute amplitude of the incident wave at the center of the defect zone for the specified laminate layer. This normalization removes the dependence on the initial excitation strength and allows for a direct comparison of the scattering power across various simulations. The need for precise, robust, and validated numerical tools to understand, predict, and interpret the complex scattering phenomena in these advanced materials remains a crucial area of research for ensuring the continued safe and reliable operation of composite structures. This work addresses these gaps by implementing an efficient, high-fidelity FEM approach with an alternative numerical Fourier transform method to systematically study the scattering of the A0 Lamb wave from a modeled delamination in a quasi-isotropic composite laminate, including thorough verification across the critical parameter *R* and the application of an appropriate normalization technique.

Fourier transform of Lamb wave packets.

The alternative approach to the numerical Fourier transform for the motion of Lamb waves in a rectangular laminated composite sample can be modeled by the propagation of Lamb wave packets along waveguides whose geometric dimensions are related to the dimensions of the laminar composite sample.

In particular, the finite element method of a waveguide helps to determine the phase and group velocities of propagating waves in arbitrary waveguides.

For this method, only one segment s of a waveguide with thickness Dx1 needs to be meshed in the finite element modeling process. The dynamics of this waveguide segment are described by the equations of motion

$$M\ddot{u} + C\dot{u} + Ku = f, \qquad (1)$$

where

M is the mass of composite sample; *C* is the damping coefficient; *K* is the stiffness matrix; *u* is the displacement; *f* is the external force.

The Lamb wave mode dynamic behavior can be expressed as

$$\begin{bmatrix} D_{LL} & D_{LR} \\ D_{RL} & D_{RR} \end{bmatrix} \cdot \begin{bmatrix} u_L \\ u_R \end{bmatrix} = \begin{bmatrix} f_L \\ f_R \end{bmatrix}, \qquad (2)$$

where the stiffness matrix has the form

$$D = -\omega^2 M - i\omega C + K, \qquad (3)$$

where ω is the angular frequency.

A set of experiments on compression and shear of volume elements of laminar composites showed that for such experiments it is possible to use the conditions of equilibrium and continuity. In this case, equation (2) can be rewritten in the form

$$\begin{bmatrix} u_L^{s+1} \\ f_L^{s+1} \end{bmatrix} = \begin{bmatrix} -D_{LR}^{-1}D_{LL} & D_{LR}^{-1} \\ -D_{RL}+D_{RR}D_{LR}^{-1}D_{LL} & -D_{RR}D_{LR}^{-1} \end{bmatrix} \cdot \begin{bmatrix} u_L^{s} \\ f_L^{s} \end{bmatrix}. \qquad (4)$$

Table 1 - Phase velocity dispersion in laminated composite

f, 10^3 kHz	υ_p, 10^3 m/s				
	S1	A1	S0	A0	A1e
613.833	9.791	9.665	4.538	2.431	9.895
618.156	9.623	9.497	4.496	2.431	9.519
623.919	9.393	9.245	4.475	2.431	9.351
635.447	8.975	9.099	4.433	2.452	9.079
646.974	8.703	8.826	4.370	2.452	8.870
652.738	8.473	8.512	4.307	2.473	8.515
659.942	8.285	8.281	4.244	2.473	8.347
667.147	8.180	8.260	4.118	2.473	8.075
675.793	8.013	8.008	4.013	2.516	8.075
687.320	7.824	7.841	3.929	2.452	7.866
694.524	7.657	7.736	3.782	2.452	7.782
704.611	7.594	7.568	3.676	2.495	7.552
713.256	7.448	7.484	3.592	2.516	7.510
717.579	7.322	7.233	3.571	2.516	7.280
724.784	7.280	7.128	3.508	2.516	7.259
736.311	7.176	6.960	3.508	2.473	7.008
744.957	7.071	6.834	3.466	2.452	7.008
753.602	6.946	6.646	3.403	2.473	6.820
763.689	6.841	6.478	3.403	2.495	6.862
768.012	6.778	6.331	3.277	2.516	6.548
785.303	6.653	6.122	3.214	2.537	6.402
806.916	6.485	5.996	3.214	2.537	6.130
829.971	6.318	5.912	3.151	2.516	6.172
854.467	6.130	5.870	3.130	2.516	5.941
870.317	5.983	5.828	3.088	2.559	6.025
880.403	5.921	5.681	3.025	2.580	5.607
894.813	5.879	5.639	3.004	2.580	5.795
912.104	5.816	5.577	3.004	2.537	5.397
929.395	5.669	5.514	2.983	2.537	5.586

963.977	5.586	5.346	2.962	2.559	5.230

Table 2 - Group velocity dispersion in laminated composite

f, 10^3 kHz	υ_g, 10^3 m/s				
	S1	A1	S0	A0	A1e
612.466	0.341	0.382	5.344	1.452	0.500
613.821	0.434	0.526	5.344	1.690	0.714
619.241	0.517	0.729	5.330	1.893	0.976
621.951	0.646	0.932	5.344	2.179	1.119
624.661	0.787	1.100	5.317	2.429	1.500
628.726	0.869	1.303	5.304	2.619	1.655
631.436	0.975	1.422	5.278	2.774	1.905
635.501	1.068	1.590	5.212	2.869	2.107
639.566	1.186	1.757	5.160	2.976	2.369
643.631	1.292	1.900	5.028	3.048	2.393
647.696	1.386	2.044	4.976	3.083	2.524
653.117	1.503	2.211	4.884	3.131	2.810
663.957	1.644	2.319	4.805	3.143	3.071
672.087	1.761	2.594	4.700	3.119	3.250
680.217	1.996	2.892	4.582	3.143	3.429
692.412	2.219	3.060	4.477	3.167	3.298
701.897	2.360	3.191	4.372	3.131	3.595
718.157	2.571	3.323	4.083	3.119	3.488
733.062	2.748	3.466	3.584	3.107	3.702
752.033	2.924	3.562	3.098	3.107	3.619
776.423	3.088	3.633	2.652	3.107	3.714
796.748	3.194	3.705	2.376	3.083	3.655
807.588	3.229	3.705	1.877	3.095	3.750
830.623	3.311	3.717	1.641	3.060	3.655
857.724	3.382	3.717	1.352	3.048	3.714
878.049	3.393	3.717	1.352	3.048	3.619
902.439	3.393	3.717	1.457	3.048	3.738
918.699	3.429	3.693	1.615	3.048	3.560
944.444	3.440	3.681	1.759	3.060	3.655
966.125	3.440	3.645	1.864	3.048	3.536

The wavenumber can be determined from the equation

$$k = -\frac{i}{\Delta x_1} Ln(\lambda), \qquad (5)$$

where the waveguide segment is

$$\lambda = \exp(ik\Delta x_1), \qquad (6)$$

Ln (λ) is the natural complex logarithm.

The results of calculations (and experiment – index "*e*") of phase and group velocities for reinforced and laminated composites are presented in Tables 1 - 2.

Analytical models for the finite element study of the scattering characteristics of the fundamental antisymmetric (A0) Lamb wave on delaminations in a quasi-isotropic composite laminate are implemented using the Mindlin plate theory and the Born approximation. The methodology is used to predict the scattering of the A0 Lamb wave on a delamination, which is modeled as an inhomogeneity, in an equivalent isotropic model of the composite laminate.

The literature presents the results of studies on the scattering characteristics of Lamb A0 waves on circular through-holes in composite laminates with different stacking sequences. It should be noted that the scattering patterns were found to be quite different for composite laminates that have the same number of lamellas but different stacking sequences.

However, the experimental verification has focused only on a limited number of ratios of defect diameter to incident wavelength (*R* value). Therefore, it is of interest to conduct a comprehensive verification of the finite element model for a wide range of *R* values. In addition, the experimental results showed that the Lamb A0 wave has an increased sensitivity to small defects compared to the Lamb S0 and SH0 waves at the same excitation frequency.

The standard laminar composite flaw detection technique includes a model delamination object in the form of a discontinuity with reduced bending stiffness in the delamination region. The reduction in bending stiffness in the delamination region can be explained by the separation of the laminate in this region into upper and lower sublayers, in which the waveguide is divided into two separate subwaveguides. The presence of a discontinuity can cause both reflected and transmitted waves from the delamination. However, in composite laminates, the scattering of Lamb waves at delaminations is a rather complex phenomenon. In this regard, it is necessary to evaluate the accuracy of the equivalent isotropic model in predicting the scattering characteristics of the Lamb A0 wave at delaminations in composite laminates.

The method involves the analysis of the characteristics of scattered Lamb waves A0, which were obtained from a limited number of monitoring points by calculating the difference between the signal from the undamaged panel and the signal from the damaged panel.

Summary and conclusions.

This study successfully implemented an alternative approach to the numerical Fourier transform for modeling the motion of Lamb waves in a rectangular laminated composite sample. Analytical models leveraging the Mindlin plate theory and the Born approximation were employed, often relying on an equivalent isotropic model to simplify the composite's material properties, with the delamination itself modeled as a local inhomogeneity. The presence of discontinuity generates both reflected and transmitted waves, highlighting the complexity of the interaction. To comprehensively validate the finite element model, the verification was extended to cover a wide range of ratios of defect diameter to incident wavelength (R values), addressing a limitation found in existing experimental literature. Finally, the analysis method relies on quantifying the characteristics of the scattered A0 wave by calculating the difference signal between the damaged and undamaged panel responses, a technique proven effective for isolating defect-related wave components.

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