



UDC 519.7:697.245.7

## MULTICRITERION OPTIMIZATION AT EVOLUTIONARY SEARCH FOR TUBULER GAS HEATERS

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**Abstract.** The task of finding an optimal solution for a known binary fuzzy choice relation is considered. An iterative algorithm is used, in which the functions of generation and selection of solutions with several branches of the evolutionary search are successively implemented. The generation function is built, mostly, regardless of the content of the task. The selection function is built using a selection procedure, depending entirely on the problem being solved. The resulting information is used to guide the search process, making it understandable for guided machine learning.

**Key words:** tubular gas heaters, binary choice relations, multicriteria optimization

### Introduction.

Solving multicriteria optimization problems requires the use of artificial intelligence methods. There is a significant number of works devoted to multicriteria optimization, for example [1,2]. These works develop the use of genetic algorithms and swarm optimization, which use a large number of modifications of these algorithms. At the same time, the use of universal approaches to solving optimization problems with binary choice relations, which is the subject of this report, has not received sufficient development.

**Main text.** Let us state the problem.

It is necessary to find a solution  $x \in \Omega$  so that  $xR_{s_1}y$  and for all  $y \in \Omega$  and so that  $xR_{s_1}y$  it is fulfilled  $xR_{s_2}y$  and  $xR_{s_3}y$ . Let us create new binary relation  $R_{SSS}$  that takes into account limitation in the form of binary relation as follows:

$$xR_{SSS}y \equiv xR_{s_1}y \wedge xR_{s_2}y \wedge xR_{s_3}y \quad (1)$$

Let us represent the original choice relation (1) in the form



$$xR_{SSS}y \equiv xR_{SS}y \wedge xR_{S3} \quad (2)$$

where

$$xR_{SS}y \equiv xR_{S1}y \wedge xR_{S2} \quad (3)$$

It is this general case (3) of multicriteria optimization that is described in [3]. According to [3] the evolutionary search is represented in form

$$X_{jk} = S^{R_s}(G(X_{jk-1})), \quad k=1, 2, \dots, j=1, 2, \dots, N_b,$$

where  $S^{R_s}(X)$  - selection function as blocking function

$$S^{R_s}(X) = \{x \in X | \forall y \in [X \setminus S^{R_s}(X)], \overline{yR_s x}\}$$

and  $X_{jk}$  – the set of selection solutions according to the binary choice relation  $R_s$  and choice function as blocking function at the iterate step  $k$  for the branch  $j$  of evolutionary search,  $N_b$  – the number of branches.

A numerical example of an evolutionary search using the boxing function is given in [3].

To jointly take into account binary choice (1), we present the evolutionary search algorithm in the form  $X_{jk} = S_{R_{S3}}(S^{R_{SS}}(G(X_{jk-1})))$ ,  $k=1, 2, \dots, j=1, 2, \dots, N_b$ ,

It is this general case (2) following our previous studies it is considered here a problem of finding a solution  $x_0 \in \Omega$  from elements  $x = \{x^1, x^2, \dots, x^n\}$ , so that  $\forall x \in \Omega$ ,  $x_0 R_s x$  where  $R_s$  - known binary relation that is a relation of non-strict preference.

We will consider the decomposition

$$X_k = \bigcup_{j=1}^{N_b} X_{jk}, \quad X_{ik} \cap X_{jk} = \emptyset, \quad i \neq j$$

where  $X_{jk}$  – the set of preferred solutions according to the binary choice relation  $R_s$  at the iterate step  $k$  for the branch  $j$  of evolutionary search,  $N_b$  – the number of branches.

Evolutionary search looks like:

$$X_{jk} = S(G(X_{jk-1})), \quad j=1, N_b, \quad k=1, 2, \dots, \quad (4)$$

where  $S(X)$  – function of choice in the form:



$$S(X) = \{x \in X \mid \forall y \in [X \setminus S(X)], xR_sy\}, \quad (5)$$

We shall assume that set  $S(X)$  contains the concrete number of elements –  $N_s$ .

For subset  $X$ ,  $X \subset \Omega$  we denote the function of generation  $G(X)$  in the form

$$G(X) = X \cup G_H(X),$$

$$G_H(X) = \{y \in \Omega \mid \exists x \in X, yR_Gx, \mu_{R_G}(x, y) > 0\}, \quad (6)$$

$R_G$  – relation  $R_G$  with attachment function  $\mu_{R_G}(x, y): \Omega \times \Omega \rightarrow [0, 1]$ .

We shall assume that set  $G(X)$  contains the concrete number of elements –  $N_E$ .

Let us denote as  $R_s^+(x)$  - upper section according to the relation  $R_s$  of the set  $\Omega$ .

$$R_s^+(x) = \{y \in \Omega \mid yR_sx\}. \quad (7)$$

Statement 1. If  $R_s^+(x)$  – upper section according to the relation  $R_s$  have the property:

$$\forall x \neq x_0, \text{mes}R_s^+(x) > 0,$$

where  $x_0$  – is  $R_s$ -optimal solution at the set  $\Omega$ ,

and the function of generation satisfies a requirement if  $x_H \in G_H(X)$  then

$$\forall x \neq x_0, P\{x_H \in R_s^+(x)\} \geq \delta > 0,$$

in this case for any  $x \in \Omega$ ,  $x \neq x_0$ , there is a number  $K$  that for any  $k \geq K$  and for all branches of the search  $j = \overline{1, N_b}$  with probability 1 a requirement will be met  $x_{jk} \subset R_s^+(x)$ , that proves convergence of the iteration process (4) with the probability that equals 1 to an  $R_s$ -optimal decision for all branches of evolutionary search  $j = \overline{1, N_b}$ .

If relation (3) is the relation is a no strictly order relation, than the evolutionary search is represented in form  $X_{jk} = S_{R_{s1}}(S_{R_{s2}}(G(X_{jk-1}))), j = 1, 2, \dots, N_B, k = 1, 2, \dots$

A specific application of the presented multicriteria optimization approach is mathematical modeling of the operation of a pellet burner for a tubular gas heater [4]. As a result of the experimental study, regression dependencies were obtained for three output parameters YA, YCO, YNOx depending on three dimensionless input



parameters. Using the methods of the theory of dimension and similarity, the modeling problem can be reduced to the modeling of 5 dimensionless parameters (complexes):

$$\Pi_1 = S_p / S, \Pi_2 = L_p / L, \Pi_3 = W / Y_A / (L / S)^2, \Pi_4 = (\alpha_{CO})^{0.5}, \Pi_5 = (\alpha_{NO_x})^{0.5}.$$

Regression dependencies in dimensionless form have the form:

$$\phi(\Pi_1, \Pi_2, \Pi_3) = 0, \Pi_4 = \psi(\Pi_1, \Pi_2, \Pi_3), \Pi_5 = \eta(\Pi_1, \Pi_2, \Pi_3).$$

The obtained dependencies for the output functions make it possible to solve problems of optimization of input parameters. Of course, it is of interest to provide conditions

$$\Pi_4 \rightarrow \min, \Pi_5 \rightarrow \min, Y_A \rightarrow \min. \quad (8)$$

Conditions (8) correspond to the formulation of the multicriteria optimization problem (1) and can be solved using the described approach. Numerical solutions of algorithm (4) showed fairly good performance of the evolutionary search.

### Summary and conclusions.

Algorithms for finding solutions with multiple binary relations have been considered. Two approaches to solving multicriteria optimization problems have been developed: 1- construction of the integral binary choice relation, which is based on the initial binary choice relations; 2 - realization of the solution search procedure in the form of sequential selection, first by the first binary choice relation, and then by the second binary choice relation from those solutions that passed the first selection. The above approaches are useful for parametric optimization of tubular gas heaters.

The results of the calculations prove that the algorithm of evolutionary search has a sufficiently good performance in solving problems of fuzzy modeling of tubular gas heaters.

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*The article was prepared as part of the  
Scientific and technical work 0122U200712 Mathematical modeling and  
multi-criteria optimization of pipeline energy systems*

Article sent: 25.05.2025

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